FAR BEYOND

MAT122

Meaning of the Derivative

Stony Brook University

Leibniz Notation

$$\boxed{f'(x) \;=\; \text{instantaneous rate of change of } f \text{ at } x.}$$

so far $f'(x)$ has been used to represent the derivative

$$f'(x) \approx \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \qquad \frac{\text{“difference in } y\text{”}}{\text{“difference in } x\text{”}}$$

$$\Big\downarrow \; \text{Leibniz Notation}$$

$$\frac{\text{derivative of } y}{\text{with respect to } x} \qquad \frac{dy}{dx} \xrightarrow[\text{written as}]{\text{can also be}} \frac{d}{dx}(y)$$

Units of a Derivative

Velocity is an example of a derivative.

position changes with respect to time

s denotes position function: $s(t)$ ex. units: $\dfrac{\overset{s}{miles}}{\underset{t}{hour}}$ "y" units / "x" units

ex. The cost C in dollars of building a house A square feet in area is given by the function $C(A)$.

What are the units of $C'(A)$? $\dfrac{dC}{dA} = \boxed{\dfrac{\text{dollars}}{\text{square foot}}}$

ex. If $q = f(p)$ gives the number of pounds of sugar produced when the price per pound is p dollars. What are the units of $\dfrac{dq}{dp}$? $\boxed{\dfrac{\text{pounds}}{\text{dollar}}}$

What is the interpretation of $\left.\dfrac{dq}{dp}\right|_{p=3} = 50$?

When the price is \$3, quantity of sugar is increasing at a rate of 50 pounds per dollar.

Interpretation

ex. The time, L, in hours that a drug stays in the system is a function of the quantity, q, administered in mg.

a. Interpret $L(10) = 6$

$L(q)$

$q = 10$ mg

$L = 6$ hours

$\boxed{\text{A dose of 10 mg lasts 6 hours.}}$

b. Write the derivative in Leibniz notation. $\boxed{\dfrac{dL}{dq}}$

"y" units "x" units

c. If $L'(10) = 0.5,$ what are the units of 0.5? $\boxed{\text{Hours per mg}}$

d. Interpret $L'(10) = 0.5$ in terms of dose and duration.

- At a dose of 10 mg, the rate of change is 0.5 hr/mg.

  \- or -

- If dose is increased by 1 mg the drug stays in the body ~1/2 hour longer.

Second Derivative

Since a derivative is a function, we can calculate **its** derivative.

"f double prime"

For the function f:

the derivative of its derivative, f', is called the <u>**second derivative**</u> and is denoted as f''.

In Leibniz notation: the derivative of the derivative, $\frac{dy}{dx}$, is $\frac{d^2y}{dx^2}$.

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Meanings of Derivatives

Increasing/Decreasing

If $f' > 0$ on an interval then f is **increasing** on that interval.

If $f' < 0$ on an interval then f is **decreasing** on that interval.

Concavity

If $f'' > 0$ on an interval then f is **concave up** on that interval.

If $f'' < 0$ on an interval then f is **concave down** on that interval.